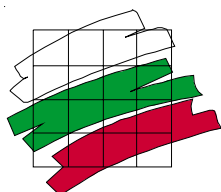# WORKING PAPER SERIES

STATISTICAL ESTIMATES OF THE DEVIATIONS FROM THE MACROECONOMIC POTENTIAL. AN APPLICATION TO THE ECONOMY OF BULGARIA

K. Ganev

Contents

1

1 Introduction

The study of the macroeconomic potential and the deviations from it is one of the comparatively new and interesting but quite controversial areas of economic analysis. In recent years the topic has gained increasing popularity because of the fact that the opportunities to stimulate economic growth through the discovery and utilization of new production resources are decreasing on a worldwide scale and therefore ways are sought to fully utilise the available ones. Besides that, the level of potential production is an important guideline for the conduct of fiscal and monetary policies, and it also serves for the determination of other important indicators, such as the natural rate of unemployment[1] and the cyclical phase of the economy.

On the other hand, there is no single widely accepted opinion on how deviations from the potential should be calculated, and there are no unanimously accepted estimation methods and models. Consequently, the various institutions, which deal with planning and forecasting, publish and sometimes make international comparisons of this indicator although the calculations have been made using completely different methodologies.

The paper aims at reviewing the statistical approaches used in estimating the deviations from the economy potential, as well as presenting some results on this indicator for the case of Bulgaria.

2 The macroeconomic potential – some clarifying definitions

The following definition of the macroeconomic potential reflects the accepted treatment of the term in literature and is based to a large extent on Okun's [18] defintition.

Definition 1 *The macroeconomic potential equals the maximum that an economy can produce without overloading itself and creating undesirable phenomena. By the term 'undesirable phenomena' we will mean basicly the acceleration of the rate of inflation and the consequences thereof.*

---

[1]The more precise term used in contemporary literature is Non-Accelerating Inflation Rate of Unemployment, NAIRU.

Definition 2 *Under 'GDP gap' we will mean the difference between the potential and the actual GDP.*

When this difference is positive, the respective economy does not utilize fully its production resources, and when it is negative, the economy is *overheated*.

3   Main groups of estimation approaches

There are several approaches to the estimation of the production potential. One of them is to use a linear or a quadratic trend. Other methods come down to the specification and the estimation of a macroeconomic production function. A third group is based to a large extent on purely statistical techniques and uses dynamic estimation with filters.

3.1   Estimation with linear and quadratic trends

The usage of estimates that extract a linear or a quadratic trend from the data series is not based on a supposed or existing structural dependence in the economy. The construction of such a model is based solely on the assumption that the economic time series are characterized with a certain degree of momentum, i.e. the 'newer' data are dependent on the 'older' data. The usage of linear trends is justified in cases where there are no major data fluctuations and mostly in cases where there are no major changes in the development of the process structure.[2]

The method amounts to estimating by OLS one of the following two equations, respectively for a linear and for a quadratic trend:

$$y_t = \beta_1 + \beta_2 \cdot t + \varepsilon_t \tag{1}$$

$$y_t = \beta_1 + \beta_2 \cdot t + \beta_3 \cdot t^2 + \varepsilon_t, \tag{2}$$

where t is time.

The estimation results from the two equations using data for Bulgaria are displayed respectively on Figure 1 and Figure 2. Logarithms of quarterly data on GDP (seasonally adjusted) at 1996 prices have been used. The period of estimation is first quarter of 1994 – second quarter of 2003.

---

[2]The so-called *structural breaks*.

4

Figure 1: GDP gap using a linear trend
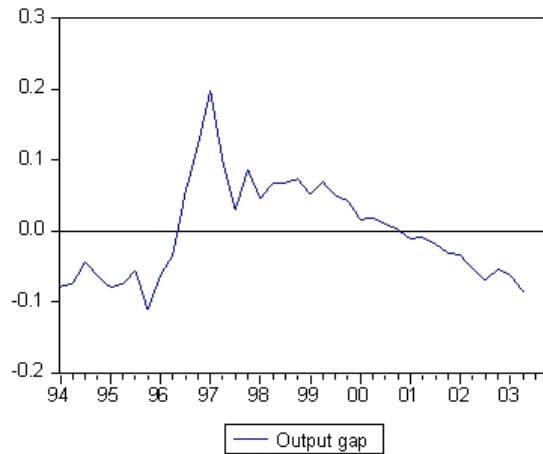
Figure 2: GDP gap using a quadratic trend
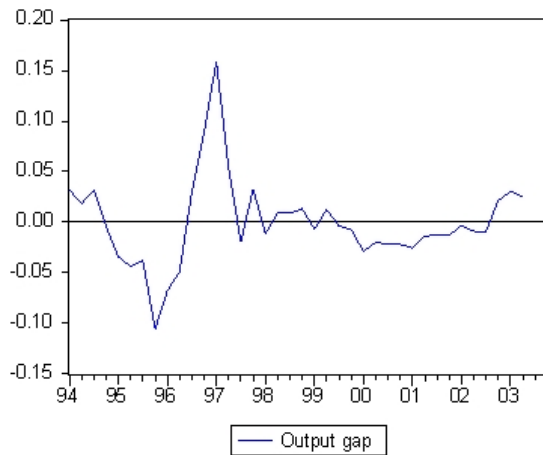
The negative numbers correspond to the so-called 'inflationary gap', i.e. to an overheated economy, and the positive – to the so-called 'recessionary gap', i.e. to occasions, in which the actual GDP is below its potential.[3]

---

[3]When in an economy a share of the production facilities are idle, the term 'inflationary gap' may be inaccurate since in such cases the the 'overheating' of the economy is not always related with inflationary processes, due to the influence of other factors acting in the reverse direction. In the case of Bulgaria such factors are the restrictions on the monetary policy (the currency board rule), the strict rules for the commercial banks and also the lack of a sufficiently strong

Since the GDP series exhibits a structural break, the linear trend is entirely unsuitable to catch the development direction (the slope of the regression line wrongly reflects the direction and the dynamics of the process). Therefore the results from the application of this model will not be commented in the conclusions section.

The quadratic trend estimate is more trustworthy since it takes into account this structural break. Although the model is quite simplified and does not rely on structural economic relations, as a first proxy to reality it provides certain opportunities to analyze this indicator.

3.2 Estimates using production functions

The usage of production functions, on the one hand, has its own merits since it allows to reflect the production structure of the economy by relating directly the production with its determinants. The disadvantages of this approach have to do mainly with the specification of the form of the production function, and with the fact that the level of technology (which is an important determinant of growth) is an unobserved component. Moreover, there are also no precise measures of the observed determinants of growth – labour, physical capital, and human capital[4], and each inaccuracy in their measurement directly reflects the value of the potential GDP and its interpretations.

The production function approach will not be commented here in detail since it requires a separate study.

3.3 Estimation using filters

Filters belong to the group of purely statistical tools. Most of them have the common feature that they are not based on any assumption as to the structure of the economy. The estimation of the economy potential is done through decomposition of output into a cyclical component and a trend. The cyclical component is assumed to be the gap, and the trend – the potential GDP.

Classical examples of such tools are the filters elaborated by Baxter and King [2], and by the Hodrick and Prescott [14]. They belong to the class of the so-called univariate filters, since they are applied to univariate time series. A characteristic feature both filters is that the quality of the estimates when

and effective demand.

[4]Education, qualifications, skills, health status, etc.

working with macroeconomic data is comparatively low, especially at the ends of the used samples, which are of the highest interest for economic policy.[5]

3.3.1 The Hodrick and Prescott filter

In a paper published in 1997 Hodrick and Prescott propose a filter for decomposing time series, and variants of this filter are widely used in the analysis of economic variables, characterized with cyclical behavior in time. The application of the filter amounts to the minimization with respect to $\overline{y}_t$ of the expression:

$$HP = \sum_{t=1}^{T}(y_t - \overline{y}_t)^2 + \lambda \sum_{t=1}^{T-1}[(\overline{y}_{t+1} - \overline{y}_t) - (\overline{y}_t - \overline{y}_{t-1})]^2, \qquad (3)$$

where y_t is the logarithm of the actual GDP, $\overline{y}_t$ is the trend, and $\lambda > 0$ is the smoothing parameter. The larger the value of this parameter, the smoother the obtained trend. For quarterly data the value of 1600 is generally preferred. The choice of this value is not theoretically substantiated, but it is often recomended by practitioners and is programmed in some statistical and econometric software packages.

In this case (and in the case of the rest of the filters) the assumption is that the observed data series representing the GDP can be decomposed into to orthogonal components – a trend and a cyclical component:

$$y_t \equiv \overline{y}_t + z_t \qquad (4)$$

In this equation $z(t)$ is the ratio of the GDP gap to the potential GDP. [6]

Actually the cyclical component is a residual – a difference between the actual (the observed) and the potential (the trend) GDP. Equation 4 is incorporated

---

[5]On critical reviews the usage of the two filters for analyzing macroeconomic time series, see for example Guay and St-Amant [12] or Harvey and Jaeger [13].

[6]Since this is a logarithm of the level of the gap, Z_t, it can be shown that from:

$$z_t = y_t - \overline{y}_t = \ln Y_t - \ln \overline{Y}_t = \ln \frac{Y_t}{\overline{Y}_t} = \ln \left(\frac{\overline{Y}_t + Z_t}{\overline{Y}_t} \right) = \ln \left(1 + \frac{Z_t}{\overline{Y}_t} \right)$$

and from:

$$z_t \approx \ln(1 + z_t) \quad \text{for small values of } z_t,$$

follows that:

$$z_t \approx \frac{Zt}{\overline{\overline{Y}}_t}.$$

in Equation 3 – the first item in the right-hand side of the equation is exactly the sum of the squared differences between the actual and the potential GDP, or, the GDP gap. The application of the filter to the respective statistical series leads to the extraction of a smooth trend, and the difference between the observed and the filtered values is the GDP gap.

The results from the application of the Hodrick and Presctott filter are displayed by Figure 3.

Figure 3: GDP gap, Hodrick and Prescott filter applied
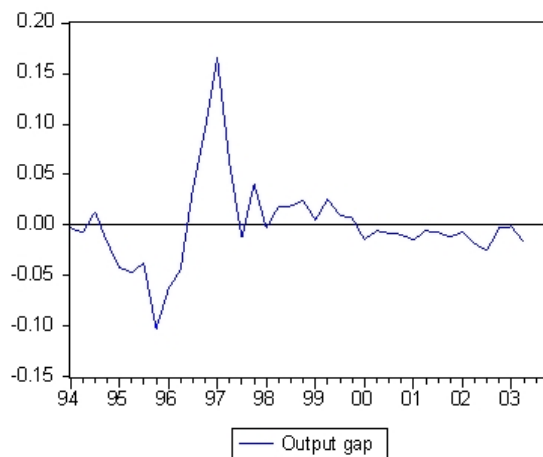
3.3.2 Band-pass filters

The business-cycle theory deals with the fluctuations of the economy in the short to medium term. It is generally accepted that the length of those fluctuations is between 6 and 32 quarters.

In the analysis of fluctuations the tools of spectral analysis are often used. According to the Spectral Representation Theorem, each data series can be decomposed into components that form its spectrum. The decomposition is done by means of the so-called *ideal band-pass filter*. However, it is a theoretical concept since it requires an infinite data series and thus, in practice, various modifications are used to get a good approximation.

The name of this class of filters comes from their nature – the aim is to isolate all components with a given frequency, which does not exceed a certain band, and all other frequencies are eliminated (filtered).

8

The ideal band-pass filter can be generally written as:

$$\widehat{y}_t = B(L)y_t = \sum_{j=-\infty}^{\infty} B_j y_{t-j}, \tag{5}$$

where:

$$
\begin{aligned}
B_j &= \frac{\sin(2\pi j/p_l) - \sin(2\pi j/p_u)}{\pi j}, \quad j \geq 1 \\
B_0 &= \frac{2p_l - 2p_u}{p_l p_u},
\end{aligned}
\tag{6}
$$

and p_l and p_u are respectively the values of the minimum and maximum period of cycle.

In practice the filters of the Baxter and King and of the Christiano and Fitzgerald are used as optimal approximations to the ideal filter.

3.3.3 The Baxter and King filter

The Baxter and King filter is a linear transformation of the data, in which the integral of the error for choosing the approximation $\widehat{B}^{p,p}$ is minimized having the constraint $\widehat{B}^{p,p}(1) = 0$:

$$\min_{\widehat{B}_j^{p,p}} \left(\int_{-\pi}^{\pi} |\widehat{B}^{p,p}(e^{-i\omega}) - B(e^{-i\omega})|^2 d\omega \right) \tag{7}$$

where:

$$
\begin{aligned}
B(e^{-iw}) &= 1, \text{ if } \omega \in (a,b) \cup (-b,-a) \\
&= 0, \text{ in the opposite,}
\end{aligned}
\tag{8}
$$

$\{(a,b) \cup (-b,-a)\}$ belongs to the interval of trend variation $(-\pi, \pi)$, and i is the imaginary unit.[7]

The results from the application of the filter are displayed in Figure 4.

3.3.4 The Christiano and Fitzgerald filter

The Christiano and Fitzgerald filter is an improved version of the Baxter and King filter. It is also a linear approximation of the ideal band-pass filter[8], in

---

[7]By Euler's formula, $e^{ix} = \cos x + i \sin x$.

[8]Note that the ideal filter itself is a linear transformation of the actual data.

Figure 4: GDP gap, Baxter and King Filter applied
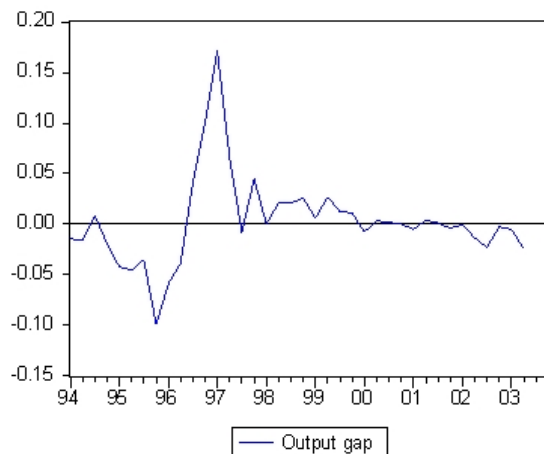
which the mean square error between the ideal filter result and the approximation is minimized. To do this, the distribution of the actual data series has to be estimated. There are two options for decomposition:

- To assume that the real time series may be characterized as a random walk. In such a case the filter is computed using the formula:

$$
\begin{aligned}
\widehat{y}_t &= B_0 x_t + B_1 x_{t+1} + \ldots + B_{T-1-t} + \widetilde{B}_{T-t} x_T + \\
&\quad + B_1 x_{t-1} + \ldots + B_{t-2} x_2 + \widetilde{B}_{t-1} x_1 \\
t &= 3, 4, \ldots, T-2,
\end{aligned}
\tag{9}
$$

where $\widetilde{B}_{T-t}$ and $\widetilde{B}_{t-1}$ are linear combinations of B_j. In our case the values of the minimum and maximum period are respectively 6 and 32.

- In the cases, where the random walk aassumption is not plausible, it is necessary to determine beforehand the stochastic form of the time series. When the series is trend- or difference-stationary, but has a non-zero mean, this mean has to be removed before carrying out the analysis.[9]

The results from the application of the filter are shown in Figure 5.

---

[9]In econometrics this procedure is known as removal of drift.

Figure 5: GDP gap, Christiano and Fitzgerald filter applied
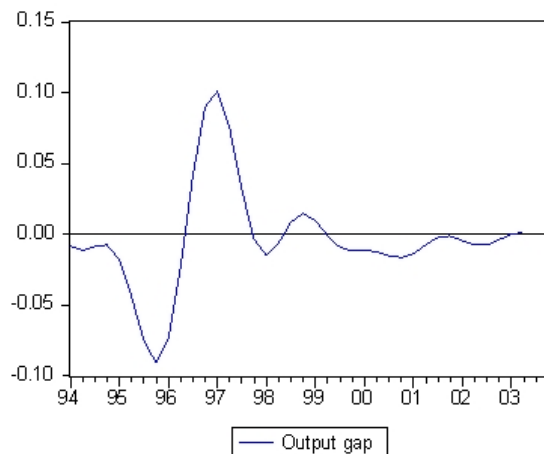
3.3.5 Univariate unobserved components models

The estimation with this approach is based on the methods proposed by Watson [23] and Clark [5]. The decomposition of the observed series (here the GDP) shown in Equation 4 is used:

$$y_t \equiv \overline{y}_t + z_t$$

Here it is assumed that each of the two components of the series (which are unobserved), develops in time according to a chosen pattern. For the potential output (the trend) it is assumed that it is a second-order random walk [10]:

$$\overline{y}_t = \mu_{t-1} + \overline{y}_{t-1} + u_t, \tag{10}$$

where $u_t \sim NID(0, \sigma_u^2)$. Besides that, the drift itself is a random walk:

$$\mu_{t+1} = \mu_t + v_{t+1}, \tag{11}$$

where $v_t \sim NID(0, \sigma_v^2)$. For the cyclical component, z_t, it is assumed that it follows an $AR(2)$-process:

$$z_t = \theta_1 z_{t-1} + \theta_2 z_{t-2} + w_t, \tag{12}$$

---

[10]The general representation of such type of processes is $(1 - L)^2 y_t = \varepsilon_t$, where L is the lag operator, and $\varepsilon_t \sim NID(0, \sigma_\varepsilon^2)$

where $w_t \sim NID(0, \sigma_w^2)$. To carry out the estimation, the system has to be rewritten in the so-called *state-space form*:

$$y_t = \begin{bmatrix} 1 & 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} \overline{y}_t \\ z_t \\ z_{t-1} \\ \mu_t \end{bmatrix} \tag{13}$$

$$\begin{bmatrix} \overline{y}_t \\ z_t \\ z_{t-1} \\ \mu_t \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & \theta_1 & \theta_2 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} \overline{y}_{t-1} \\ z_{t-1} \\ z_{t-2} \\ \mu_{t-1} \end{bmatrix} + \begin{bmatrix} u_t \\ v_t \\ 0 \\ w_t \end{bmatrix} \tag{14}$$

The model is estimated by the application of the Kalman and Bucy filter to maximize the likelihood function:

$$\ln L = -\frac{N}{2} \ln 2\pi - \frac{1}{2} \sum_{t=1}^{T} |\ln F_t| - \frac{1}{2} \sum_{t=1}^{T} q_t' F_t^{-1} q_t, \tag{15}$$

where T is the number of observations, q_t are the forecast errors, and F_t is mean-square error matrix.

Besides Watson and Clark, a similar method is used for example by Harvey and Jaeger [13].

Figure 6: GDP gap, univariate UC model applied

12
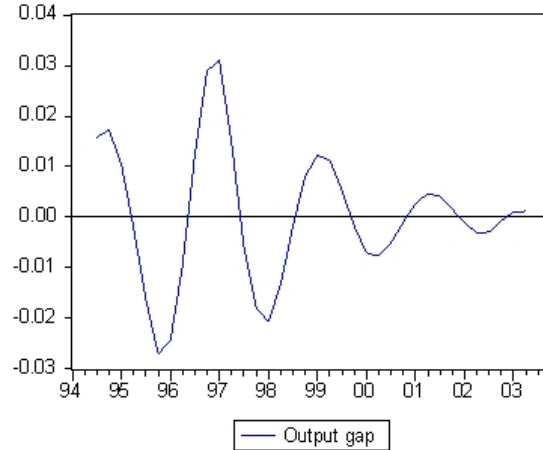
3.4 Estimation with mixed models

Because of the described disadvantages of the univariate filters, many attempts have been made to combine the filtering approach with some elements of economic structure. This type of models forms the so-called multivariate filters. Additional information relating to the economic structure can be introduced through variants of the Philips curve, through relations treating Okun's law, through measures of the capacity utilization, etc. Approaches of this class can be found for example in Giorno et al. [11], St-Amant and van Norden [21], Bautista [1] , etc.

These practical applications use mainly the approaches of Kuttner [15] and Gerlach and Smets [9]. The estimation is carried out with algorithms, which use the maximum likelihood method and the Kalman and Bucy filter.

3.4.1 The Kuttner approach

Kuttner complements the model given in the system of equations 13 and 14 with a Philips curve equation, in which the changes in the inflation rate are related to the changes in the GDP gap:

$$\Delta \pi_t = \eta_1 + \eta_2 \Delta y_t + \eta_3 z_t + \gamma(L)\varepsilon_t, \tag{16}$$

where $\gamma(L) = 1 + \gamma_1 L + \ldots + \gamma_q L^q, \quad L^s x_t = x_{t-s}$.

Besides changes in inflation, for example the changes in the level of unemployment or another suitable variable can be used as dependent variable. Depending on the specific case, the lag length in the residuals is chosen. Depending on the lag length and due to the certain freedom in the choice of the dependent variable, the model can have different (functional) forms. That is why it is called also "the generalized Kuttner model". For example, if $s = 0$, $\gamma(L) = 1$, i.e. the residuals are white noise with a zero mean.

3.4.2 The Gerlach and Smets approach

This approach is very similar to the Kuttner approach, with the exception that the changes in GDP are not present as a regressor in the second (complementary) equation):

$$\Delta \pi_t = \eta_1 + \eta_2 z_t + \gamma(L)\varepsilon_t, \tag{17}$$

3.4.3 An application of a specification of the generalized Kuttner model to Bulgarian data

The usage of inflation as a dependent variable does not lead to sensible results when applying two-equation approach. That is why the changes in the level of unemployment are used as a dependent variable. The reasoning behind this is as follows: since in a period of economic recovery the rate of unemployment is expected to rise, and vice versa (in periods of recession), it is logical to look for a relationship between the cyclical movement of the GDP and the uemployment dynamics.

The usage of the changes in unemployment rather than the levels of unemployment is dictated by the fact that in Kuttner's model the dependent variable has to be stationary, which in the case is achieved by first-differencing.

To the available statistical data on the economy of Bulgaria we apply the following specification of the Kuttner model (the estimation is carried out with $\mathrm{GAP}^{\circledR}$ software taking into account the methods described in Maravall and Planas [16]):

$$\left|\begin{array}{l} y_t = \overline{y}_t + z_t \\ \overline{y}_t = \mu_{t-1} + \overline{y}_{t-1} + u_t \\ \mu_{t+1} = \mu_t + v_{t+1} \\ z_t = \theta_1 z_{t-1} + \theta_2 z_{t-2} + w_t \\ \Delta un_t = \eta_1 + \eta_2 \Delta y_t + \eta_3 z_t + \eta_4 D_t + \sum_{i=0}^{3} \phi_i \varepsilon_{t-i} \end{array}\right. \tag{18}$$

where un_t is the unemployment level at time t, D_t is a dummy variable[11], and $\phi_0 = 1$.

4 Conclusions

Commenting the results for the Bulgarian economy may be problematic in a sense, given the fact that according to the business survey of the National Statistical Institute a large share of the production capacity (up to 40%) is idle. At first glance we could infer that a certain share of the capacity is idle indeed, for example, due to a weak internal demand. However, it does not make sense to assume that the enterpreneurs invest in almost twice larger capacity than they

---

[11]Its values are 1 for the period 4th quarter of 1996 – 2nd quarter of 1997 and zero for all other time points. This variable has been used to model the unusual increase of unemployment, respectively decrease of output in those three quarters.

Figure 7: GDP gap, bivariate UC model applied
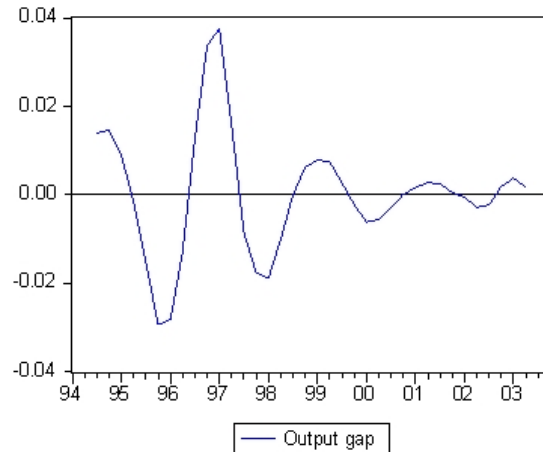
Figure 8: GDP gap, all methods (excluding linear trend)
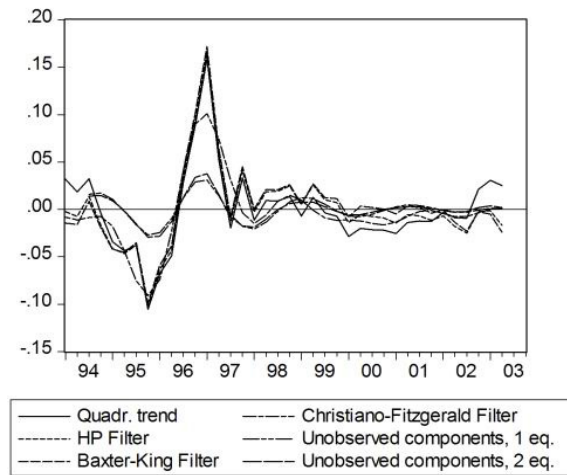
could practically use – i.e. such an assumption would mean that either they cannot foresee rationally the business environment at all, or have slack financial resources, which they spend on unsubstantiated investment purchases. It is obvious that such hypotheses are unrealistic. Therefore the macroeconomic potential concept should be viewed from a slightly different perspective. Having in mind the definitions used in literature it should be concluded that with this high level of physical capital inactivity and with the still high levels of the rate

15

of unemployment the actual output should also be at a much lower level than its potential (i.e. GDP gap level should be comparable in value with the level of idle capacities).

On the other hand, however, it is questionable whether the slack physical capacity (as much as it really exists) would contribute to economic growth if engaged in production. It is highly probable that a (large) share of the physical capital of the manufacturing enterprises is economically inefficient, or even entirely inoperable.[12] Its alleged state to a large extent is determined by whether it has been created with greenfield investments, through privatization of assets, built 10, 15, or more years ago, with a combination of both, etc. It is also possible that the information obtained through the surveys is not reliable due to an unrealistic estimation of the interviewed persons on how much a given enterprise can produce when loaded normally.[13]

Analogously, a certain share of the unemployed persons either have qualifications, which do not comply with the characteristics demanded by the employers, or their knowledge and skills are "outdated" and would not be applicable in a modern type of production, without additional qualification or re-qualification. Moreover, after more than 10 years of structural reforms a pool of long-term unemployed has formed in the group of unemployed, and those people either quit the labor force when they lose their hope of finding a job, or, despite continuing formally to look for a job, have lost their skills, working habits and connections to their once practiced profession.

Having in mind the above cosiderations it would be reasonable to assume that the potential output is the output, which can be produced with a complete loading of the economically efficient and operable production resources. These resources can also be idle for certain periods of time, and this would determine the cyclical behavior of the economy.

The results obtained from the various methods show differences in magnitude and in some occasions also in the direction (the sign) of the deviations from the potential. Nevertheless the estimates are close to each other, at least regarding the dynamics of the GDP gap, which can be verified by the correlations presented

---

[12]This may be due to moral depreciation, to lost markets that used to exist to the time of the collapse of the Council for Mutual Economic Assistance (CMEA), to evolutions in the consumption characteristics in the country, to the existence of more efficient producers or importers, etc.)

[13]Here maybe the analogy with the normal load of the economy working with production factors with certain qualities and its overloading when overheated, is appropriate. Of course, all these considerations are mere assumptions.

in Annex 1, Table 2. What can be inferred with sufficient confidence is that at the end of the period the economy is almost at its potential (the deviations range from some tenths of the percentage to a maximum of 2.5%). Furthermore, the analysis of the results from the conduct of stabilization policies launched after the financial and economic crisis at the end of 1996 and the beginning of 1997, points out that the business cycle has been substantially smoothened, i.e. a much more easily foresseable economic environment is in place. As a comparison, in the first half of the period under review the estimates of the cyclical behavior show large deviations: in the period up to mid-1996 a gap involving inflationary pressures is observed, and after that there is a collapse in the economic system and respectively a decrease of output to a level under its potential.

As for the future development of the indicator, it would be most appropriate to use as an econometric base for forecasting the estimated unobserved components models. The Hodrick and Prescott filter, the Baxter and King filter, and the Christiano and Fitzgerald filter do not lead to very reliable results for the latest observation points of the studied period, which some of the authors of the filters also admit. As a consequence it can be expected that the quality of forecasts will also deteriorate. The estimates with a linear or a quadratic trend could be used to evaluate the future development of the trend (the potential GDP). The deviations from it, however, cannot be forecasted directly, since they are a pure residual variable with an unknown data generating process. To do this we would have to build a separate econometric model, with which to forecast the dynamics of the actual GDP. This, however, would stultify the trend estimation done here.

Forecasting with the unobservable components methods is favorable from an econometric point of view, since it allows for a direct generation of forecasts of the GDP gap using the estimated $AR(2)$-equation. Only the estimated values of the parameters θ_1 and θ_2 are necessary, as well as the values of the gap for the preceding two periods. Of course, since in our case we have forecasting with quarterly data, the accumulation of the forecast errors for a relatively short period will be faster, compared for example with an annual data model, and the forecasted values for a longer term would be increasingly inaccurate.

The estimated values of θ_1 and θ_2 for the single-equation modelare respectively 1.3967 and -0.8424, and for the two-equation model they are 1.2988 and -0.7444. In both cases the stationarity of the process is obvious since $|\theta_1 + \theta_2| < 1$. This, combined with the values of the estimated gap, means that the fluctuations in the forecast period will decrease and the economy will operate closely to its potential, i.e. the actual GDP growth will approximately coincide with the growth of the potential.

Since, as already mentionedabove, the existence of an inflationary pressure depends on the sign and the magnitude of the GDP, it can be inferred that the forecasted cyclical development will not be a source of inflationary pressures. This means that the increase of the price level (if any) should be attributed to other factors but not to an overheated economy.

References

[1] C. C. Bautista. Estimates of output gaps in four Southeast Asian Countries. *Economics Letters*, 80(3):365–371, 2003.

[2] M. Baxter and R. King. Measuring business cycles: Approximate band-pass filters for economic series. *NBER Working Paper No. 5022*, 1995.

[3] L. J. Christiano and T. J. Fitzgerald. The business cycle: It's still a puzzle. *Economic Perspectives*, 1998.

[4] L. J. Christiano and T. J. Fitzgerald. The band-pass filter. *NBER Working Paper No. 7257*, 1999.

[5] P. Clark. Trend reversion in real output and unemployment. *Journal of Econometrics*, 40:15–32, 1989.

[6] P. Conway and B. Hunt. Estimating potential output: a semi-structural approach. *Reserve Bank of New Zealand Discussion Paper G97/9*, 1997.

[7] R. F. Engle and D. L. McFadden, editors. *Handbook of econometrics*, volume 4. Elsevier, 1999.

[8] P. H. Franses. *Time Series Models for Busines and Economic Forecasting*. Cambridge University Press, 2000.

[9] S. Gerlach and F. Smets. Ouput gaps and inflation: Unobservable-components estimates for the G-7 countries. *Bank for International Settlements Working Paper (unpublished)*, 1997.

[10] S. Gerlach and M. S. Yiu. Estimating output gaps in Asia: A cross-country study. *Journal of the Japanese and International Economies (forthcoming)*, 2003.

[11] C. Giorno, P. Richardson, D. Roseveare, and P. van den Noord. Estimating potential output, output gaps and structural budget balances. *OECD Working Paper*, 1995.

[12] A. Guay and P. St-Amant. Do the Hodrick-Prescott and Baxter-King filters provide a good approximation of business cycles? *Montreal Center for Research on Economic Fluctuations and Employment Working Paper No. 53*, 1997.

[13] A. C. Harvey and A. Jaeger. Detrending, stylized facts and the business-cycle. *Journal of Applied Econometrics*, pages 231–47, 1993.

[14] R. J. Hodrick and E. Prescott. Post-war U.S. business cycles: An empirical investigation. *Journal of Money, Credit and Banking*, 29(1), 1997.

[15] K. N. Kuttner. Estimating potential output as a latent variable. *Journal of Business and Economic Statistics*, 12:361–368, 1994.

[16] A. Maravall and C. Planas. Estimation errors and the specification of unobserved component models. *Journal of Econometrics*, 1999.

[17] P. S. Maybeck. *Stochastic Models, Estimation and Control*, volume 1. Academic Press, 1979.

[18] A. Okun. Potential GDP: Its measurement and significance. *Proceedings of the American Statistical Association*, 1962.

[19] A. Orphanides and S. van Norden. The reliability of output gap estimates in real time. *The Review of Economics and Statistics*, 1999.

[20] T. Proietti, A. Musso, and T. Westermann. Estimating potential output and the output gap for the euro area. *European Univeristy Institute Working Paper No. 2002/9*, 2002.

[21] P. St-Amant and S. van Norden. Measurement of the output gap: A discussion of recent research at the Bank of Canada. *Bank of Canada Technical Report No. 79*, 1997.

[22] P. van den Noord. The size and role of automatic fiscal stabilizers in the 1990s and beyond. *OECD Working Paper No. 230*, 2000.

[23] M. Watson. Univariate detrending methods with stochastic trends. *Journal of Monetary Economics*, 18:49–75, 1986.

Annexes

Annex 1

Results and statistics

Table 1: GDP gap estimates obtained with the various methods

Year and quarter	QTR	HP	BK	CF	UC, 1eq.	UC, 2eq.
1994-1	0.032	-0.002	-0.014	-0.008	NA	NA
1994-2	0.019	-0.007	-0.016	-0.011	NA	NA
1994-3	0.032	0.014	0.009	-0.009	0.016	0.014
1994-4	-0.004	-0.017	-0.019	-0.007	0.017	0.015
1995-1	-0.034	-0.042	-0.042	-0.018	0.011	0.009
1995-2	-0.044	-0.047	-0.045	-0.044	-0.002	-0.001
1995-3	-0.038	-0.038	-0.035	-0.075	-0.016	-0.015
1995-4	-0.105	-0.102	-0.099	-0.091	-0.027	-0.029
1996-1	-0.068	-0.063	-0.059	-0.074	-0.024	-0.028
1996-2	-0.049	-0.043	-0.039	-0.023	-0.010	-0.013
1996-3	0.030	0.036	0.041	0.041	0.012	0.013
1996-4	0.088	0.095	0.101	0.090	0.029	0.034
1997-1	0.159	0.166	0.172	0.101	0.031	0.038
1997-2	0.053	0.060	0.065	0.075	0.015	0.016
1997-3	-0.020	-0.012	-0.008	0.031	-0.006	-0.008
1997-4	0.033	0.041	0.045	-0.004	-0.018	-0.018
1998-1	-0.011	-0.003	0.000	-0.015	-0.021	-0.019
1998-2	0.009	0.019	0.021	-0.006	-0.013	-0.010
1998-3	0.009	0.019	0.021	0.008	-0.002	0.000
1998-4	0.014	0.025	0.026	0.015	0.008	0.006
1999-1	-0.007	0.005	0.006	0.010	0.012	0.008
1999-2	0.013	0.026	0.027	-0.001	0.011	0.008
1999-3	-0.004	0.010	0.012	-0.009	0.005	0.003
1999-4	-0.007	0.007	0.011	-0.012	-0.002	-0.002
2000-1	-0.029	-0.014	-0.008	-0.011	-0.007	-0.006
2000-2	-0.020	-0.006	0.003	-0.012	-0.008	-0.006
2000-3	-0.021	-0.008	0.002	-0.015	-0.005	-0.003

2000-4	-0.022	-0.009	0.000	-0.016	-0.001	0.000
2001-1	-0.025	-0.014	-0.005	-0.014	0.003	0.002
2001-2	-0.014	-0.005	0.004	-0.007	0.005	0.003
2001-3	-0.012	-0.007	0.001	-0.002	0.004	0.003
2001-4	-0.013	-0.011	-0.004	-0.001	0.002	0.001
2002-1	-0.003	-0.007	0.000	-0.004	-0.001	0.000
2002-2	-0.009	-0.018	-0.014	-0.008	-0.003	-0.003
2002-3	-0.009	-0.025	-0.023	-0.007	-0.003	-0.002
2002-4	0.021	-0.002	-0.003	-0.004	-0.001	0.002
2003-1	0.031	-0.001	-0.005	0.000	0.001	0.004
2003-2	0.025	-0.016	-0.024	0.001	0.001	0.002

QTR - quadratic trend

HP - Hodrick and Prescott filter

BK - Baxter and King filter

CF - Christiano and Fitzgerald filter

UC - Unobservable component model

Table 2: Correlations among the estimates obtained with the various methods

	QTR	HP	BK	CF	UC, 1eq.	UC, 2eq.
QTR	1.000	0.952	0.929	0.877	0.717	0.793
HP	0.952	1.000	0.996	0.890	0.683	0.753
BK	0.929	0.996	1.000	0.886	0.663	0.734
CF	0.877	0.890	0.886	1.000	0.767	0.813
UC, 1eq.	0.717	0.683	0.663	0.767	1.000	0.984
UC, 2eq.	0.793	0.753	0.734	0.813	0.984	1.000

Table 3: Statistics, linear trend model

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	12.986	0.023244	558.6884	0.0000
TREND	0.002605	0.001039	2.506961	0.0168
R-squared	0.148631	Mean dependent var		13.03679
Adjusted R-squared	0.124982	S.D. dependent var		0.075081
S.E. of regression	0.070232	Akaike info criterion		-2.422816
Sum squared resid	0.177574	Schwarz criterion		-2.336627
Log likelihood	48.0335	F-statistic		6.284854
Durbin-Watson stat	0.283339	Prob(F-statistic)		0.016831

Table 4: Statistics, quadratic trend model

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	13.11505	0.02288	573.2009	0.0000
TREND	-0.016753	0.002706	-6.19215	0.0000
TREND-SQUARED	0.000496	6.73E-05	7.377213	0.0000
R-squared	0.666777	Mean dependent var		13.03679
Adjusted R-squared	0.647736	S.D. dependent var		0.075081
S.E. of regression	0.044562	Akaike info criterion		-3.308217
Sum squared resid	0.069502	Schwarz criterion		-3.178934
Log likelihood	65.85613	F-statistic		35.01736
Durbin-Watson stat	0.695024	Prob(F-statistic)		0.0000

Table 5: Statistics, single-equation UC model

	Coefficient	S.E.	t-stat
AR1:	1.3967	0.2401	5.818
AR2:	-0.8424	0.15	-5.617
Trend innov var:	1.16E-03		
Trend slope var:	4.94E-06		
Cycle innov var:	4.84E-05		
-2*log-likelihood:	-132.1564		
Ljung-Box stat. Q(4)	0.4941	p-value	0.9741

Table 6: Statistics, two-equation UC model

	Coefficient	S.E.	t-stat
First equation			
AR1:	1.2988	0.3163	4.1067
AR2:	-0.7444	0.2212	-3.3658
Trend innov var:	1.11E-03		
Trend slope var:	5.53E-06		
Cycle innov var:	9.38E-05		
Ljung-Box stat. Q(4)	0.4851	p-value	0.9749
Second equation			
Intercept:	-0.0702	0.0073	-9.577
Gamma - lag 1:	0.0076	0.0582	0.1307
Beta - lag 0:	0		
MA1:	0.5743	0.0056	101.7072
MA2:	0.4694	0.0091	51.5951
MA3:	0.3023	0.0048	63.6026
Innovation var:	3.31E-04		
Corr innov cycle-eqn1	1.32E-03		
Exog 1:	1.0000	0.0163	61.2348
-2*log-likelihood:	42002.904		
Ljung-Box stat. Q(4)	0.1032	p-value	0.9987
R-squared (uncentered)	0.491		

Annex 2

Representation of the potential GDP as a second-order random walk

We can write:

$$\overline{y}_t - \overline{y}_{t-1} - u_t = \mu_{t-1} \quad (a)$$
$$\mu_{t-1} = \mu_t - v_t \quad\quad\quad (b)$$

From (a) follows that the next statement is also true:

$$\mu_t = \overline{y}_{t+1} - \overline{y}_t - u_{t+1} \quad (c)$$

We substitute the obtained result in (b):

$$\mu_{t-1} = \overline{y}_{t+1} - \overline{y}_t - u_{t+1} - v_t \quad (d)$$

From (a) and (d) follows that:

$$\overline{y}_t - \overline{y}_{t-1} - u_t = \overline{y}_{t+1} - \overline{y}_t - u_{t+1} - v_t \Leftrightarrow$$
$$\Leftrightarrow \overline{y}_{t+1} = 2\overline{y}_t - \overline{y}_{t-1} + \xi_{t+1} \Leftrightarrow$$
$$\Leftrightarrow (1 - L)^2 \overline{y}_{t+1} = \xi_{t+1},$$

where $\xi_{t+1} = u_{t+1} - u_t + v_t$.

AEAF PAPER SERIES

POLICY PAPER SERIES

1. On the Pace of the Economic Reform and Economic Policy Objectives by the End of 1991 (June, 1991).

2. R. Avramov (ed.) – Economic Stabilization in Bulgaria in 1992 (June, 1992).

3. Bulgaria – The Economic Situation and Outlook. The Status of the Reform Process (May, 1993).

4. R. Avramov, K. Guenov – The Rebirth of Capitalism in Bulgaria (October 1994).

WORKING PAPER SERIES

1. M. Nenova-Amar – The 1991 Budget and Some Policy Implications in 1992. (March, 1992).

2. M. Zhecheva, R. Avramov, V. Chavdarov – Inflation and the Interest Rate in 1991. (March, 1992).

3. Stan. Barzashki – Employment and Unemployment in the Process of Stabilization. (March,1992).

4. N. Gueorguiev, N. Gospodinov – Monetary Policy: Mechanisms and Outcomes. (March, 1992).

5. R. Injova – Privatization in Bulgaria. (July, 1992).

6. M. Zhecheva, N. Mileva – Price Controls and Inflation in Bulgaria, 1991-1992. (November 1992).

7. K. Genov – Monetary Policy in 1992: Instruments and Results. (April, 1993).

8. M. Nenova-Amar – Wage Controls: the Bulgarian Experience in 1991/1992 . (April 1993).

9. N. Gueorguiev – Some Tests of Random Walk Hypothesis for Bulgarian Foreign Exchange Rates. (August 1993).

10. L. Dimitrov – Unemployment in Bulgaria, 1991-1993. (July 1994).

11. M. Zhecheva – Households and Financial Flows in the Bulgarian Economy. (September 1994).

12. N. Mileva – Agriculture and Agrarian Policy in 1994. (March 1995).

13. M. Nenova, Alfredo Canavese – State-owned Enterprises' Behaviour in Transition Economies and Inflation (The Case of Bulgaria 1991-1995). (June 1996).

14. R. Krustev – Free Trade Versus Protectionism. Promoting Competition in Bulgaria as an Anti-Inflationary Factor. (1998)

15. L. Dimitrov – Budget Constraints of Bulgarian Enterprises, 1996-1997. (1999)

16. G. Chukalev – Inflation under a currency board arrangement. Factor analysis. (2000)

17. T. Manchev – Management of Capital Inflows in Bulgaria. (2002)

18. G. Chukalev – The Balassa – Samuelson Effect in the Bulgarian Economy. (2002)

BUSINESS SURVEY SERIES

1. Monthly Business Surveys – Since October 1991

2. Quarterly Business Surveys (since I Quarter 1992 till IV Quarter 1993)

3. Semi-Annual Reports (since July 1995)

4. Annual Reports on the State of the Bulgarian Economy (1991, 1992, 1993, 1994, 1995, 1996, 1997, 1998, 1999, 2000, 2001)

❍   ❍   ❍

Economic Transition in Bulgaria. Edited by Roumen Avramov and Ventsislav Antonov. Sofia, 1994.

Bulgaria on the Road to Europe. A collection of papers on the macroeconomic developments in Bulgaria and the country's association with the European Union. (September 1996)